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Journal of Sound and Vibration 295 (2006) 378–387
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Master–slave chaos synchronization criteria for the horizontal platform systems via linear state error feedback control

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Received 29 July 2005; received in revised form 7 January 2006; accepted 14 January 2006
Available online 29 March 2006

Abstract

Global chaos synchronization of two identical non-autonomous horizontal platform systems coupled by linear state error feedback controller is investigated. The sufficient criteria for global chaos synchronization are deduced based on the stability theory of linear time-varied systems and Lyapunov's direct method, of which, the criteria related to general coupling matrix are first proved and applied to derive the ones related to some special coupling matrices. In the examples, the coupling strengths are designed by the obtained criteria and the appearances of chaos synchronization are verified. It is analytically and numerically examined that the synchronization criteria based on Lyapunov's direct method are sharper than the criteria based on the stability theory of linear time-varied systems.

1. Introduction

Synchronization of chaotic systems has been widely investigated [1–13] since the early work by Pecora and Carroll [4]. In the past decade, the research on chaos synchronization has intensively focused on the autonomous chaotic systems, e.g. Lur'e systems [5–9], Lorenz system [10,11], Chen [12] and Rössler systems [13], etc. Recently, many non-autonomous chaotic systems have been discovered in engineering and life science [14–20], and their synchronization has been discussed in Refs. [15–17,19].

Ge et al. [16] numerically verified that two identical horizontal platform systems coupled, respectively, by linear, sinusoidal and exponential state error feedback controllers can achieve chaos synchronization. The coupling strengths resulting in chaos synchronization are detected according to negativity of all Lyapunov exponents of the driven system. However, the condition that all Lyapunov exponents are negative has been confirmed to be only necessary but not sufficient for chaos synchronization [1,4].

In this paper, using the stability theory on linear time-varied systems and Lyapunov's direct method [21], we deduce the sufficient criterion for global chaos synchronization of two identical horizontal platform systems coupled by linear state error feedback controller. The algebraic criteria related to general and some special

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.01.038

coupling matrices are derived. These sufficient criteria can be applied to directly design the coupling strength resulting in the synchronization.

The structure of this paper is as follows. In Section 2, we present a master–slave synchronization scheme for non-autonomous horizontal platform systems, and derive the relevant error system. In Section 3, the sufficient criteria for global chaos synchronization are proved based on the stability theory on linear time-varied systems and Lyapunov's direct method. In Section 4, the examples are illustrated to verify the theoretical results. Finally, the concluding remark is described in Section 5.

2. Synchronization problem and error system

In 2003, Ge et al. studied the chaotic phenomena of the following non-autonomous horizontal platform system with an accelerometer [16]

$$A\ddot{y} + D\dot{y} + rg\sin y - \frac{3g}{R}(B - C)\cos y \sin y = F\cos\omega t, \tag{1}$$

where y denotes the rotation of the platform relative to the earth, A, B and C are, respectively, the inertia moment of the platform for three axes which penetrate the mass center of the platform, D is the damping coefficient, r the proportional constant of the accelerometer, g the acceleration constant of gravity, R the radius of the Earth, and $F\cos\omega t$ harmonic torque. More details of this model can be seen in Ref. [16].

The horizontal platform system (1) can be represented as the non-autonomous form:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -ax_2 - b\sin x_1 + l\cos x_1 \sin x_1 + h\cos\omega t, \end{aligned} \tag{2}$$

where

$$a = \frac{D}{A} > 0, \quad \mathrm{b} = \frac{rg}{A} > 0, \quad l = \frac{3g}{RA}(B - C), \quad h = \frac{F}{A} > 0.$$

Let $x = (x_1, x_2)^{\mathrm{T}} \in R^2$. The vector form of the system (2) is

$$\dot{x} = Ax + f(x) + m(t) \tag{3}$$

with

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -a \end{pmatrix}, \quad f(x) = \begin{pmatrix} 0 \\ -b\sin x_1 + l\cos x_1 \sin x_1 \end{pmatrix}, \quad m(t) = \begin{pmatrix} 0 \\ h\cos\omega t \end{pmatrix}.$$

Now we consider a master–slave synchronization scheme for two identical horizontal platform systems coupled by a linear state error feedback controller $u(t) = K(x - z)$ with the constant coupling matrix $K \in R^{2\times 2}$ as follows:

$$\begin{aligned} &M : \dot{x} = Ax + f(x) + m(t), \\ &S : \dot{z} = Az + f(z) + m(t) + u(t), \\ &C : u(t) = K(x - z), \end{aligned} \tag{4}$$

where the state variables of the slave system $z = (z_1, z_2)^{\mathrm{T}} \in R^2$.

Defining the error variable $e = x - z$, we can obtain the dynamical error system

$$\dot{e} = (A - K)e + f(x) - f(z) = (A - K)e + Q(t)e = (A - K + Q(t))e, \tag{5}$$

where

$$Q = \begin{pmatrix} 0 & 0 \\ q(t) & 0 \end{pmatrix},$$

$$q(t) = \frac{-b(\sin x_1 - \sin z_1) + l(\sin x_1 \cos x_1 - \sin z_1 \cos z_1)}{x_1 - z_1}. \tag{6}$$

Our aim is to select the coupling matrix K such that the trajectories $x(t)$ and $z(t)$ of master and slave systems, wherever the choice of the initial states $x(0)$ and $z(0)$, satisfy

$$\lim_{t\to\infty} ||x(t) - z(t)|| = 0, \tag{7}$$

where $||\cdot||$ denotes the Euclidean norm of the vector.

Obviously, $e = 0$ is an equilibrium point of the error system (5). Chaos synchronization in the sense of Eq. (7) is equivalent to global asymptotic stability of the linear time-varied error system (5) at the origin.

3. Sufficient criteria for global chaos synchronization

The following lemma will be applied to prove the main theorems of the paper.

Lemma 1. *For $q(t)$ defined by* (6)*, the inequality*

$$|q(t)| \leqslant b + |l| \tag{8}$$

holds.

Proof. By the differential mean-value theorem, we have

$$\sin x_1 - \sin z_1 = \cos \xi (x_1 - z_1),\ \xi \in [x_1, z_1] \text{ or } [z_1, x_1], \text{ and}$$
$$\sin 2x_1 - \sin 2z_1 = 2\cos(2\eta)(x_1 - z_1), \eta \in [x_1, z_1] \text{ or } [z_1, x_1].$$

So,

$$q(t) = \frac{-b(\sin x_1 - \sin z_1) + l/2(\sin 2x_1 - \sin 2z_1)}{x_1 - z_1} = -b\cos\xi + l\cos(2\eta),$$

and thus inequality (8) holds. □

We now utilize the stability theory on linear time-varied systems to derive the sufficient criterion for global chaos synchronization in the sense of Eq. (7). The following theorem is related to general control matrix

$$K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \in R^{2\times 2}. \tag{9}$$

Theorem 1. *If the coupling matrix* (9) *is selected such that*

$$k_{11} + k_{22} + a > 0, \tag{10}$$

$$4k_{11}(k_{22} + a) > (|1 - k_{12} - k_{21}| + b + |l|)^2, \tag{11}$$

then the master–slave scheme (4) *achieves global chaos synchronization.*

Proof. According to the stability theory on linear time-varied systems, we know the linear time-varied system (5) is globally asymptotically stable at the origin, if

$$A - K + Q + (A - K + Q)^{\mathrm{T}} = \begin{pmatrix} -2k_{11} & 1 + q(t) - (k_{12} + k_{21}) \\ 1 + q(t) - (k_{12} + k_{21}) & -2(k_{22} + a) \end{pmatrix} \tag{12}$$

is negative definite.

The eigenvalues λ of the matrix $(A-K+Q)+(A-K+Q)^{\mathrm{T}}$ satisfy

$$\begin{aligned}&|\lambda I_2-(A-K+Q)-(A-K+Q)^{\mathrm{T}}|\\&\quad=\lambda^2+2(k_{11}+k_{22}+a)\lambda+4k_{11}(a+k_{22})-(1+q-k_{12}-k_{21})^2=0.\end{aligned}$$

Hence, from Routh–Hurwitz criterion of the matrix theory [22], it follows that matrix (12) is negative definite if and only if

$$k_{11}+k_{22}+a>0,$$

$$\begin{vmatrix}2(k_{11}+k_{22}+a) & 1\\ 0 & 4k_{11}(a+k_{22})-(1+q-k_{12}-k_{21})^2\end{vmatrix}>0,$$

or, equivalently,

$$\begin{aligned}&k_{11}+k_{22}+a>0,\\&4k_{11}(a+k_{22})-(1+q-k_{12}-k_{21})^2>0.\end{aligned}\tag{13}$$

By Lemma 1, we have

$$|1+q-k_{12}-k_{21}|\leqslant|1-k_{12}-k_{21}|+|q|\leqslant|1-k_{12}-k_{21}|+b+|l|.$$

Inequalities (13) then hold on conditions (10) and (11). □

Based on the above theorem, some synchronization criteria with respect to simple controller may be obtained, which are represented in the following corollaries.

Letting inequalities (10) and (11) be with $k_{11}=k_1$, $k_{22}=k_2$, and $k_{12}=k_{21}=0$, we have

Corollary 1. *If the coupling matrix defined by* $K=\mathrm{diag}\{k_1,k_2\}$ *is selected such that*

$$k_1+k_2+a>0,\tag{14}$$

$$4k_1(k_2+a)>(1+b+|l|)^2,\tag{15}$$

then the master–slave scheme (4) *achieves global chaos synchronization.*

Further supposing $k_1=k_2=k$, we obtain

Corollary 2. *If the coupling matrix* $K=kI_2$ *is selected such that*

$$k>\frac{-a+\sqrt{a^2+(1+b+|l|)^2}}{2}>0,\tag{16}$$

then the master–slave scheme (4) *achieves global chaos synchronization.*

Taking $k_1=k$ *and* $k_2=0$ *in inequalities* (14) and (15), *we can prove*

Corollary 3. *If the coupling matrix defined by* $K=\mathrm{diag}\{k,0\}$ *is selected such that*

$$k>\frac{(1+b+|l|)^2}{4a},\tag{17}$$

then the master–slave scheme (4) *achieves global chaos synchronization.*

Remark 1. The above synchronization criteria are based on the linear matrix inequality (12), which may be conservative. In the following, we will give some more flexible (sharper) synchronization criteria using Lyapunov's direct method.

Theorem 2. *If there exists a symmetric positive definite matrix* $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix} \in R^{2\times 2}$ *and the coupling matrix* $K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \in R^{2\times 2}$ *such that for any* $t>0$ *the matrix*

$$[A - K + Q(t)]^{\mathrm{T}} P + P[A - K + Q(t)] \tag{18}$$

is *negative definite, then the master–slave scheme* (4) *achieves global chaos synchronization.*

Proof. Take a quadratic Lyapunov function

$$V(e) = e^{\mathrm{T}} P e$$

with the symmetric positive definite matrix $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix}$. The derivative of $V(e)$ with respect to time along the system trajectory (5) is

$$\dot{V}(e) = \dot{e}^{\mathrm{T}} P e + e^{\mathrm{T}} P \dot{e} = e^{\mathrm{T}}[(A - K + Q)^{\mathrm{T}} P + P(A - K + Q)]e.$$

$\dot{V}(e)<0$ if matrix (18) is negative definite for any $t>0$.

Hence, the linear time-varied system (5) is globally asymptotically stable on condition that matrix (18) is negative definite. □

Theorem 3. *If the symmetric positive definite matrix* $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix}$ *and coupling matrix* $K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}$ *are selected such that*

$$\Omega_1 = -k_{11}p_{11} - k_{21}p_{12} + |p_{12}|(b + |l|) < 0, \tag{19}$$

$$\Omega_2 = p_{12}(1 - k_{12}) - p_{22}(a + k_{22}) < 0, \tag{20}$$

$$4\Omega_1\Omega_2 > [|p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) - p_{22}k_{21}| + p_{22}(b + |l|)]^2, \tag{21}$$

then inequality (18) *holds, hence the master–slave scheme* (4) *achieves global chaos synchronization.*

Proof. We first have

$$\begin{aligned}
&(A - K + Q)^{\mathrm{T}} P + P(A - K + Q) \\
&= \begin{pmatrix} -2k_{11}p_{11} + 2p_{12}(q - k_{21}) & p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) + p_{22}(q - k_{21}) \\ p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) + p_{22}(q - k_{21}) & 2p_{12}(1 - k_{12}) - 2p_{22}(a + k_{22}) \end{pmatrix}.
\end{aligned}$$

The above symmetric matrix is negative definite if and only if

$$-2k_{11}p_{11} + 2p_{12}(q - k_{21}) < 0, \tag{22}$$

$$2p_{12}(1 - k_{12}) - 2p_{22}(a + k_{22}) < 0, \tag{23}$$

$$\begin{aligned}
&[-2k_{11}p_{11} + 2p_{12}(q - k_{21})][2p_{12}(1 - k_{12}) - 2p_{22}(a + k_{22})] \\
&\quad - [p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) + p_{22}(q - k_{21})]^2 > 0.
\end{aligned} \tag{24}$$

Since the matrix P is positive definite, we have $p_{22}>0$. It follows from Lemma 1 that

$$-2k_{11}p_{11}+2p_{12}(q-k_{21})\leqslant 2\Omega_1,$$

$$\begin{aligned}&\left|p_{11}(1-k_{12})-p_{12}(k_{11}+k_{22}+a)+p_{22}(q-k_{21})\right|\\&\quad\leqslant\left|p_{11}(1-k_{12})-p_{12}(k_{11}+k_{22}+a)-p_{22}k_{21}\right|+p_{22}(b+|l|).\end{aligned}$$

Hence for any $t>0$, inequalities (22)–(24) hold if inequalities (19)–(21) are satisfied. □

The following corollaries are with respect to the simplified controllers.

Corollary 4. *If the coupling matrix defined by* $K=\mathrm{diag}\{k_1,k_2\}$ *and the symmetric positive definite matrix* $P=\begin{pmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{pmatrix}$ *are selected such that*

$$k_1>\frac{|p_{12}|(b+|l|)}{p_{11}},\tag{25}$$

$$k_2>\frac{p_{12}-ap_{22}}{p_{22}},\tag{26}$$

$$\begin{aligned}&4[k_1p_{11}-|p_{12}|(b+|l|)][k_2p_{22}-p_{12}+ap_{22}]\\&\quad>[|p_{11}-p_{12}(k_1+k_2+a)|+p_{22}(b+|l|)]^2\end{aligned}\tag{27}$$

then the master–slave scheme (4) *achieves global chaos synchronization.*

Proof. It follows from the positive definite P that $p_{11}>0$ and $p_{22}>0$. Inequalities (25)–(29) can be obtained according to inequalities (19)–(21) with $k_{11}=k_1, k_{22}=k_2$ and $k_{12}=k_{21}=0$. □

Corollary 5. *If the coupling matrix defined by* $K=kI_2$ *and the symmetric positive definite matrix* $P=\begin{pmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{pmatrix}$ *are selected such that*

$$k>\max\left\{\frac{|p_{12}|(b+|l|)}{p_{11}},\ \frac{p_{12}-ap_{22}}{p_{22}}\right\}\geqslant 0,\tag{28}$$

$$\begin{aligned}&4(p_{11}p_{22}-p_{12}^2)k^2-4k[2p_{22}|p_{12}|(b+|l|)+p_{11}(p_{12}-ap_{22})\\&\quad-|p_{12}(p_{11}-ap_{12})|]+4|p_{12}|(b+|l|)(p_{12}-ap_{22})\\&\quad-[|p_{11}-ap_{12}|+p_{22}(b+|l|)]^2>0\end{aligned}\tag{29}$$

then the master–slave scheme (4) *achieves global chaos synchronization.*

Proof. It is easy to get

$$[|p_{11}-p_{12}(k_1+k_2+a)|+p_{22}(b+|l|)]^2\leqslant[|p_{11}-ap_{12}|+|p_{12}(k_1+k_2)|+p_{22}(b+|l|)]^2.\tag{30}$$

Hence, inequalities (28) and (29) can be obtained by letting $k_1=k_2=k$ in inequalities (25)–(27) and (30). Again since $p_{11}p_{22}-p_{12}^2>0$, the solution k to inequality (29) exists. □

Remark 2. We may as well select $p_{12}=0$ and $p_{11}=p_{22}(b+|l|)>0$ to construct a symmetric positive definite matrix $P=p_{22}\begin{pmatrix}b+|l| & 0\\ 0 & 1\end{pmatrix}$. According to the matrix, the following synchronization criterion can be obtained by means of inequalities (28) and (29)

$$K = kI_2, \quad k > \frac{\sqrt{a^2 + 4(b + |l|)} - a}{2} > 0. \tag{31}$$

Comparing Eq. (31) with Eq. (16), we know that the lower bound of inequality (31) is less than that of inequality (16). This shows the synchronization criterion (31) is sharper than (16) and the flexibility of the inequalities (28) and (29).

It follows from inequalities (25)–(27) and (30) with $k_1 = k$ and $k_2 = 0$ that

Corollary 6. *If the coupling matrix defined by* $K = \mathrm{diag}\{k, 0\}$ *and the symmetric positive definite matrix* $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix}$ *are selected such that*

$$k > \frac{|p_{12}|(b + |l|)}{p_{11}}, \tag{32}$$

$$p_{12} - ap_{22} < 0, \tag{33}$$

$$\begin{aligned} &p_{12}^2 k^2 + 2k\,[|p_{12}(p_{11} - ap_{12})| + |p_{12}|p_{22}(b + |l|) - 2(ap_{22} - p_{12})p_{11}] \\ &\quad + 4|p_{12}|(ap_{22} - p_{12})(b + |l|) + [|p_{11} - ap_{12}| + p_{22}(b + |l|)]^2 < 0, \end{aligned} \tag{34}$$

then the master–slave scheme (4) *achieves global chaos synchronization.*

Remark 3. If we take $p_{12} = 0$ and $p_{11} = p_{22}(b + |l|) > 0$, then the following synchronization criterion is obtained based on inequalities (32)–(34):

$$K = \mathrm{diag}\{k, 0\}, \quad k > \frac{b + |l|}{a}, \tag{35}$$

which is obviously sharper than Eq. (17).

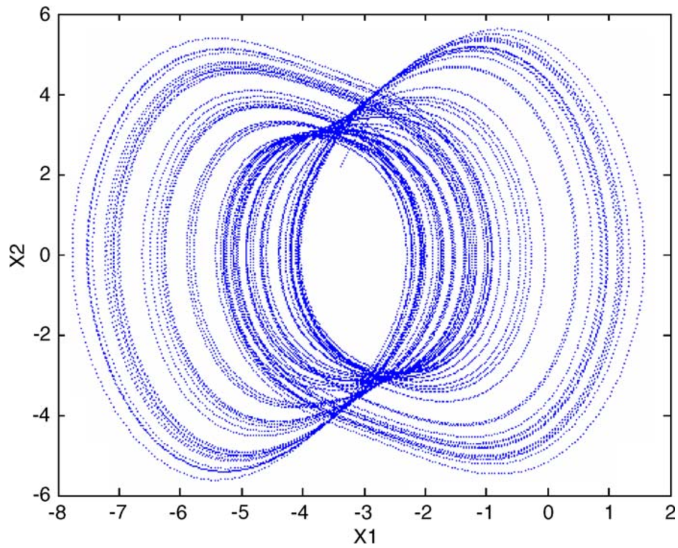


Fig. 1. Chaotic attractor of the non-autonomous horizontal platform system with $A = 0.3$, $B = 0.5$, $C = 0.2$, $D = 0.4$, $r = 0.11559633$, $R = 6,378,000$, $g = 9.8$, $F = 3.4$, $\omega = 1.8$.

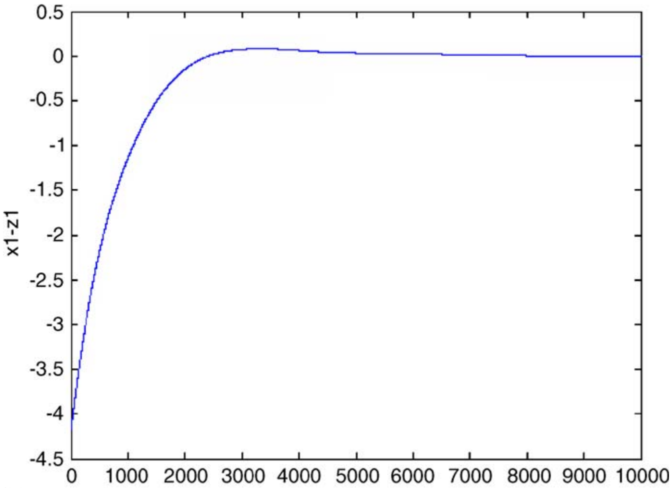
4. Examples

We take the parameters of the horizontal platform system $A = 0.3$, $B = 0.5$, $C = 0.2$, $D = 0.4$, $r = 0.11559633$, $R = 6,378,000$, $g = 9.8$, $F = 3.4$, $\omega = 1.8$. The initial conditions of the master and slave systems are $(x_1(0), x_2(x_2(0)) = (-3.4, 2.1)$ and $(z_1(0), z_2(0)) = (0.78, -2.9)$, respectively, which are freely chosen. The simulation shows the trajectory of the master system has the double scroll attractor, which implies chaotic behavior, as shown in Fig. 1.

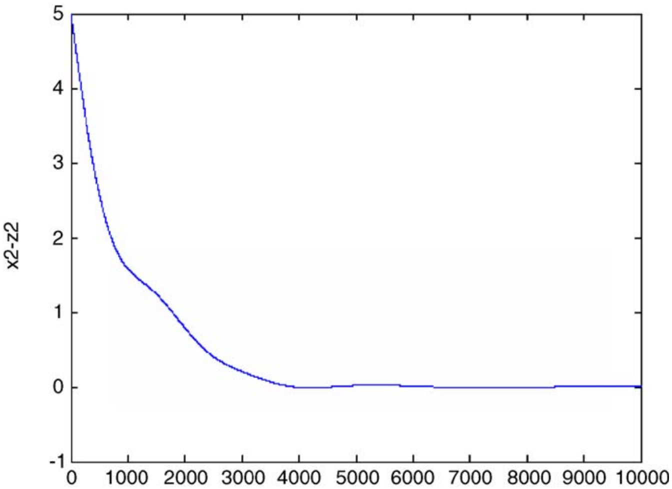
Let the coupling matrix $K = kI_2$. The synchronization conditions, $k > 1.813$ and $k > 1.388$, are obtained by means of the algebraic criteria (16) and (31), respectively.

Select the coupling matrix $K = \text{diag}\{k, 0\}$. It is solved that the synchronization conditions, $k > 4.278$ and $k > 2.833$, by the algebraic criteria (17) and (35), respectively.

The above numerical calculations reveal that the lower bounds of the coupling strength (k) resulting from the synchronization criteria (31) and (35) are less than the ones resulting from the synchronization criteria (16) and (17), respectively. Therefore, the synchronization criteria (31) and (35) based on Lyapunov's direct method are, respectively, sharper than the criteria (16) and (17) based on the stability theory of linear time-varied systems.



(a)



(b)

Fig. 2. Chaos synchronization of two non-autonomous horizontal platform systems by the controller $K = \text{diag}\{1.4, 1.4\}$.

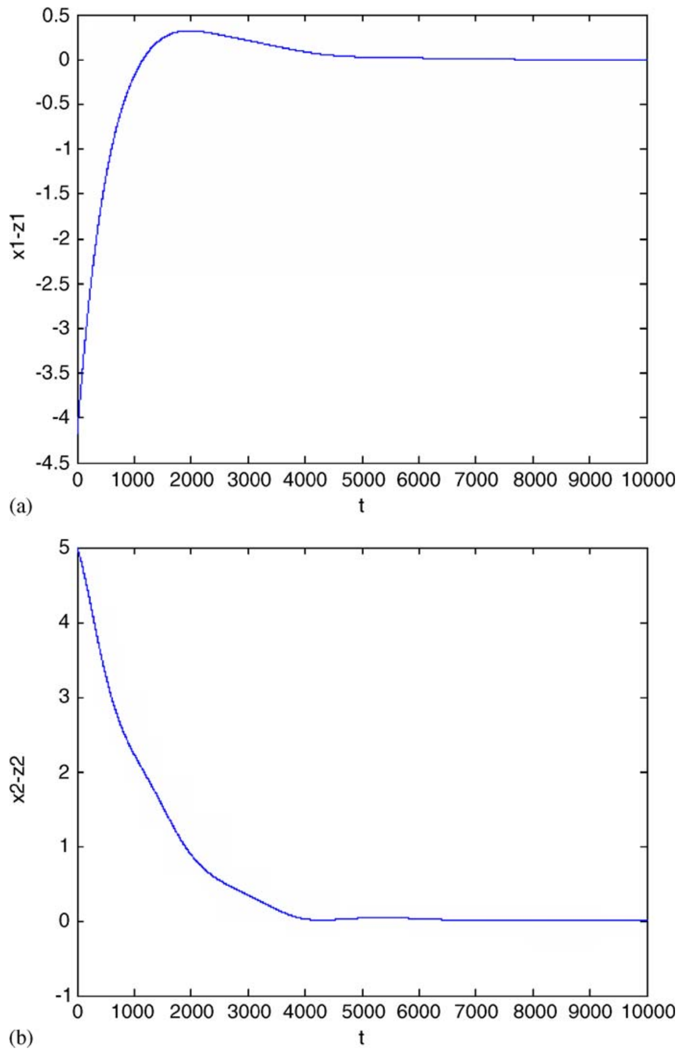


Fig. 3. Chaos synchronization of two non-autonomous horizontal platform systems by the controller $K = \text{diag}\{2.85, 0\}$.

Taking $k = 1.4$ for the coupling matrix $K = kI_2$ and $k = 2.85$ for the coupling matrix $K = \text{diag}\{k, 0\}$, we illustrate the evolutions of the error variable $e = x - z$ in Figs. 2 and 3, respectively. The results imply that the slave system is driven to asymptotically follow the chaotic dynamics of the master one. Because of the free choice of the initial states of the master and slave systems, the synchronization is considered to be global.

5. Conclusion

A method of studying global chaos synchronization of the non-autonomous horizontal platform systems coupled by linear state error feedback controller was proposed in this paper. The sufficient criteria for global chaos synchronization with respect to the general coupling matrix were deduced based on the stability theory of linear time-varied systems and Lyapunov's direct method. These criteria were applied to derive the special criteria related to some simple form of the coupling matrices. The examples were illustrated to verify the theoretical results. It is examined that the synchronization criteria based on Lyapunov's direct method are sharper than that based on the stability theory of linear time-varied systems by means of the analytic and numerical methods. We expect this work is extended to further research on chaos synchronization of the horizontal platform systems coupled by sinusoidal or exponential state error feedback controller, as in Ref. [16].

Acknowledgment

Research is supported by China Postdoctoral Science Foundation under Grant Nos. 2005038177 and 2004036517.

References

[1] S. Boccaletti, J. Kurths, G. Osipov, D.L. Valladares, C.S. Zhou, The synchronization of chaotic systems, *Physics Reports* 366 (2002) 1–101.
[2] J.A.K. Suykens, M.E. Yalccin, J. Vandewalle, Chaotic systems synchronization, in: G.R. Chen, X. Yu (Eds.), *Chaos and Bifurcation Control: Theory and Application, Lecture Notes in Control and Information Sciences*, Vol. 292, Springer, Hong Kong, 2004, pp. 117–135.
[3] G.R. Chen, X. Dong, *From Chaos to Order: Methodologies, Perspectives and Applications*, World Scientific, Singapore, 1998.
[4] L.M. Pecora, T.L. Carroll, Synchronization in chaotic systems, *Physical Review Letters* 64 (1990) 821–824.
[5] P.F. Curran, L.O. Chua, Absolute stability theory and the synchronization problem, *International Journal of Bifurcation and Chaos* 7 (1997) 1375–1382.
[6] J.A.K. Suykens, P.F. Curran, L.O. Chua, Master–slave synchronization using dynamic output feedback, *International Journal of Bifurcation and Chaos* 7 (1997) 671–679.
[7] J.A.K. Suykens, J. Vandewalle, Master–slave synchronization of Lur'e systems, *International Journal of Bifurcation and Chaos* 7 (1997) 665–669.
[8] X.F. Wu, Y. Zhao, Frequency domain criterion for chaos synchronization of Lur'e systems via state feedback control, *International Journal of Bifurcation and Chaos* 15 (2005) 1445–1454.
[9] S. Bowong, F.M.M. Kakmeni, R. Koina, A new synchronization principle for a class of Lur'e systems with applications in secure communication, *International Journal of Bifurcation and Chaos* 14 (2004) 2477–2491.
[10] Y.G. Yu, S.C. Zhang, Global synchronization of three coupled chaotic systems with ring connection, *Chaos, Solitons and Fractals* 24 (2005) 1233–1242.
[11] J.H. Lü, T.S. Zhou, S.C. Zhang, Chaos synchronization between linearly coupled chaotic systems, *Chaos, Solitons and Fractals* 14 (2002) 529–541.
[12] E.M. Elabbasy, H.N. Agiza, M.M. El-Dessoky, Synchronization of modified Chen system, *International Journal of Bifurcation and Chaos* 14 (2004) 3969–3979.
[13] Z. Li, C.Z. Han, S.J. Shi, Modification for synchronization of Rössler and Chen chaotic systems, *Physics Letters A* 301 (2002) 224–230.
[14] Y. Lei, W. Xu, Y. Xu, T. Fang, Chaos control by harmonic excitation with proper random phase, *Chaos, Solitons and Fractals* 21 (2004) 1175–1181.
[15] H.K. Chen, Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping, *Journal of Sound and Vibration* 255 (2002) 719–740.
[16] Z.M. Ge, T.C. Yu, Y.S. Chen, Chaos synchronization of a horizontal platform system, *Journal of Sound and Vibration* 268 (2003) 731–749.
[17] Z.M. Ge, J.K. Lee, Chaos synchronization and parameter identification for gyroscope system, *Applied Mathematics and Computation* 163 (2005) 667–682.
[18] L.J. Chen, J.B. Li, Chaotic behavior and subharmonic bifurcations for a rotating pendulum equation, *International Journal of Bifurcation and Chaos* 14 (2004) 3477–3488.
[19] R. Yamapi, P. Woafo, Dynamics and synchronization of coupled self-sustained electromechanical devices, *Journal of Sound and Vibration* 285 (2005) 1151–1170.
[20] K. Grudziński, J.J. Zebrowski, Modelling cardiac pacemakers with relaxation oscillators, *Physica A* 336 (2004) 153–162.
[21] J.E. Slotine, W.P. Li, *Applied Nonlinear Control*, China Machine Press, Beijing, 2004.
[22] R.A. Horn, C.R. Johnson, *Topics in Matrix Analysis*, Cambridge University Press, Cambridge, 1991.